<u>Step 1</u> – Get the absolute value expression by itself **on the left side** of the inequality.

Ex. $|4x + 7| < 19$ → This is ready for Step 2! 

Ex. $14 \leq |x - 5|$ → This is **not ready** for Step 2! Let's just reverse the whole thing, and we'll get…

$|x - 5| \geq 14$ which is ready for Step 2.

Ex. $3|x - 2| + 4 > 22$ → This is **not ready** for Step 2! Subtract 4 so

$3|x - 2| > 18$ Divide by 3 to get

$|x - 2| > 6$ Now it is ready for Step 2.

Ex. $-11 \leq -|2x + 1|$ → This is **not ready** for Step 2! Multiply by -1.

$11 \geq |2x + 1|$ Reverse the whole thing to get

$|2x + 1| \leq 11$ Now it is ready for Step 2.

<u>Step 2</u> – Check the number **on the right side** of the inequality.

- If it is positive or 0, then you have more solving to do and you can go to **Step 3**.

- If it is negative, then you need to do some thinking…
  a. Is your inequality like $|x| > -$  (or $|x| \geq -$ ) ? The answer must be **all real numbers** (All real numbers have absolute values that are positive or zero – and that's greater than any negative number!).
  b. Is your inequality like $|x| < -$  (or $|x| \leq -$ ) ? The answer must be **no solution** (if you are a positive number or zero, there's no way you can be less than or equal to a negative number!).

Let's see some examples…

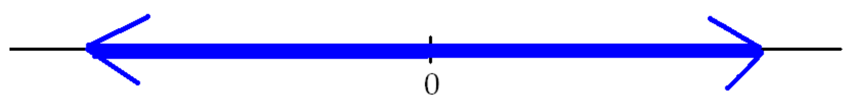
Ex. $\left|\frac{2}{3}x - 8\right| > -2$ → All absolute values are > -2, so the solution is **all real numbers**.

Ex. $|10x + 15| \leq -20$ → It is impossible for an absolute value to be ≤ -20, so there is **no solution**.

<u>Step 3</u>- Is the number on the right side of your inequality 0? If it is, we will check that out here. If it is a positive number, go to **Step 4**.

- Is your inequality like $|x| \geq 0$? Aren't all absolute values ≥ 0? It doesn't really matter what is inside the absolute value expression – when you are done taking its absolute value, you will have a number that is ≥ 0. So the solution is **all real numbers**.

Ex. $\left|\frac{x}{3} - 27\right| \geq 0$ → It doesn't matter what you put in for *x*. When you get around to finding an absolute value, your answer will be ≥ 0. The solution is **all real numbers**.



- Is your inequality like $|x| > 0$? The only place you have a problem is where your expression actually does $= 0$. Find out when that happens and eliminate it from your answer.

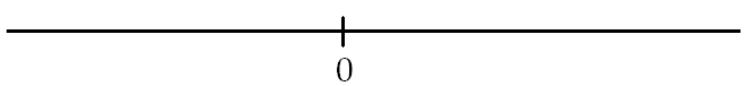
Ex. $\left|\frac{x}{4} - 12\right| > 0$ → Your only problem is when $\frac{x}{4} - 12 = 0$. Solve for *x* to discover that this happens when $x = 48$. That's the only number *x* <u>can't</u> be. Write your answer as $\boldsymbol{x \neq 48}$ or **all real numbers except** $\boldsymbol{x = 48}$.

- Is your inequality like $|x| \leq 0$? The only place it really works is when your expression *does* $= 0$. Find out when that happens and that is your solution.

Ex. $\left|\frac{x}{5} + 20\right| \leq 0$ → The left side will never end up < 0, but it is possible to $= 0$. Solve $\frac{x}{5} + 20 = 0$ to determine when that happens, and it's only when $x = -100$. Write your answer as $\boldsymbol{x = -100}$**.**

- Is your inequality like $|x| < 0$? This will *never* happen – absolute values are never negative and those are the only real numbers less than 0.

Ex. $\left|\frac{x}{2} - 8\right| < 0$ → You can't have an absolute value that is < 0, so your answer will be **no solution**.



<u>Step 4</u> – If you're here, then you have the absolute value expression by itself on the **left** side and positive number by itself on the **right** side. Let's analyze the appearance of your inequality…

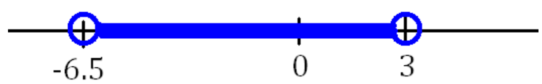
- Is your inequality like $|x| <$  (or $|x| \leq$ ) ? We will turn this into a conjunction sANDwich! Make it look like this…

$-$  $< x <$  (or $-$  $\leq x \leq$ )

Ex. $|4x + 7| < 19$ → this will become $-19 < 4x + 7 < 19$

$$-26 < 4x < 12$$

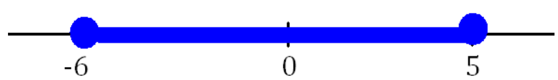
$$\boldsymbol{-\frac{13}{2} < x < 3}$$



Ex. $|2x + 1| \leq 11$ → this will become $-11 \leq 2x + 1 \leq 11$

$$-12 \leq 2x \leq 10$$

$$\boldsymbol{-6 \leq x \leq 5}$$



Any numbers you pick in the blue-colored parts of the graph will make the inequality true.

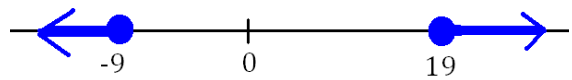
- Is your inequality like $|x| >$  (or $|x| \geq$ ) ? We will turn this into a disjunction – two parts with "or" between them. Make it look like this…

$x >$  OR $x < -$  (or $x \geq$  OR $x \leq -$ )

Ex. $|x - 5| \geq 14$ → this becomes $x - 5 \geq 14$ or $x - 5 \leq -14$

$x \geq 19$ or $x \leq -9$

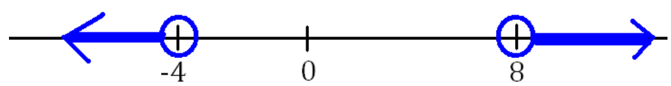
$\boldsymbol{x \leq -9}$ **or** $\boldsymbol{x \geq 19}$



Ex. $|x - 2| > 6$ → this becomes $x - 2 > 6$ or $x - 2 < -6$

$x > 8$ or $x < -4$

$\boldsymbol{x < -4}$ **or** $\boldsymbol{x > 8}$



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