In Algebra I, we learned how to take two points and figure out the equation of a line that contained those two points [use the two points to calculate the slope, put the slope and one of the points into $y - y_1 = m(x - x_1)$, etc.].

In Algebra II, we've been looking at quadratic equations & functions (they are in the shape of parabolas, not straight lines). Sometimes we can take two points and figure out an equation (or a function) that contains the two points.

<u>When we know the vertex and one other point…</u>

<u>Example 1</u> Write a quadratic function (in vertex form) that has a vertex at $(3, -6)$ and contains the point $(1, 2)$.

Step 1 – Use the vertex to plug in for (h, k) in vertex form.

$$y - k = a(x - h)^2$$
$$y - (-6) = a(x - (3))^2$$
$$y + 6 = a(x - 3)^2$$

Step 2 – To solve for a, plug in the other point for x and y.

$$y + 6 = a(x - 3)^2$$
$$(2) + 6 = a((1) - 3)^2$$
$$8 = a(-2)^2$$
$$8 = a(4)$$
$$2 = a$$

Step 3 – Put it all together in vertex form.

$$\boldsymbol{y + 6 = 2(x - 3)^2}$$

<u>When we know two roots/solutions/x-intercepts…</u>

We will use the following equation

$$\boldsymbol{x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0}$$

<u>Example 2</u> Roots/solutions are 5 and -4. Write a quadratic equation.

Step 1 – Calculate the sum of the roots

$$\text{sum of roots} = 5 + (-4) = \mathbf{1}$$

Step 2 – Calculate the product of the roots

$$\text{product of roots} = (5)(-4) = \mathbf{-20}$$

Step 3 – Plug in to $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2 - (1)x + (-20) = 0$$
$$x^2 - x - 20 = 0$$

Step 4 – No fractions are allowed, so multiply to take care of fractions (if necessary)

Not necessary in this case

$$\boldsymbol{x^2 - x - 20 = 0}$$

Notice that if we factor $\boldsymbol{x^2 - x - 20 = 0}$, we will get $\boldsymbol{(x-5)(x+4) = 0}$. That would mean $\boldsymbol{x = 5, -4}$ … the very solutions we had at the start!

Example 3 Roots/solutions are $5 + 3i$ and $5 - 3i$. Write a quadratic equation.

Step 1 – Calculate the sum of the roots

$$\text{sum of roots} = 5 + 3i + 5 - 3i = \mathbf{10}$$

Step 2 – Calculate the product of the roots

$$\text{product of roots} = (5 + 3i)(5 - 3i)$$
$$\text{product of roots} = 25 - 15i + 15i - 9i^2$$
$$\text{product of roots} = 25 - 9(-1) = 25 + 9 = \mathbf{34}$$

Step 3 – Plug in to $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2 - (10)x + (34) = 0$$
$$x^2 - 10x + 34 = 0$$

Step 4 – No fractions are allowed, so multiply to take care of fractions (if necessary)

Not necessary in this case

$$\boldsymbol{x^2 - 10x + 34 = 0}$$

Example 4 Roots/solutions are $\frac{3+\sqrt{11}}{5}$ and $\frac{3-\sqrt{11}}{5}$. Write a quadratic equation.

Step 1 – Calculate the sum of the roots

$$\text{sum of roots} = \frac{3+\sqrt{11}}{5} + \frac{3-\sqrt{11}}{5}$$
$$\text{sum of roots} = \frac{3+\sqrt{11}+3-\sqrt{11}}{5} = \mathbf{\frac{6}{5}}$$

Step 2 – Calculate the product of the roots

$$\text{product of roots} = \left(\frac{3+\sqrt{11}}{5}\right)\left(\frac{3-\sqrt{11}}{5}\right)$$
$$\text{product of roots} = \frac{(3+\sqrt{11})(3-\sqrt{11})}{(5)(5)}$$
$$\text{product of roots} = \frac{9-3\sqrt{11}+3\sqrt{11}-11}{25} = -\mathbf{\frac{2}{25}}$$

Step 3 – Plug in to $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2 - \left(\frac{6}{5}\right)x + \left(-\frac{2}{25}\right) = 0$$
$$x^2 - \frac{6}{5}x - \frac{2}{25} = 0$$

Step 4 – No fractions are allowed, so multiply to take care of fractions (if necessary)

We will multiply everything by **25**.

$$\mathbf{25}\left(x^2 - \frac{6}{5}x - \frac{2}{25} = 0\right)$$

$$\boxed{\mathbf{25x^2 - 30x - 2 = 0}}$$