Our first special pattern is the *difference of squares*. We can use this when we have two perfect square terms with a $-$ sign between them.

$$a^2 - b^2 = (a+b)(a-b)$$

Example 1

Factor $w^2 - 9$

The square root of 9 is 3, so we could rewrite this as $w^2 - 3^2$.

$$w^2 - 9$$
$$(w)^2 - (3)^2$$ ← It is OK to do this step in your head
$$\mathbf{(w+3)(w-3)}$$

Example 2

Factor $16m^2 - 81$

The square root of $16m^2$ is $4m$ and the square root of 81 is 9.

$$16m^2 - 81$$
$$(4m)^2 - (9)^2$$ ← It is OK to do this step in your head
$$\mathbf{(4m+9)(4m-9)}$$

Example 3

Factor $12x^2 - 75$

Look for the GCF first- it looks like it is 3, so $12x^2 - 75 = 3(4x^2 - 25)$

The square root of $4x^2$ is $2x$ and the square root of 25 is 5.

$$12x^2 - 75$$
$$3(4x^2 - 25)$$
$$3[(2x)^2 - (5)^2]$$ ← It is OK to do this step in your head
$$\mathbf{3(2x+5)(2x-5)}$$

Our next special pattern is the *square of a binomial*. This is useful when the 1st and 3rd terms are both perfect squares.

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (\boldsymbol{a} + \boldsymbol{b})^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (\boldsymbol{a} - \boldsymbol{b})^2$$

Example 4

Factor $x^2 + 10x + 25$

Step 1- Are the 1st and 3rd terms both perfect squares?

Yes – The square root of x^2 is x and the square root of 25 is 5.

Step 2- Does the 2nd term = 2 • square root of 1st term • square root of 2nd term?

Yes – 2 • x • 5 = $10x$, which is the 2nd term.

Step 3- Look at the first operator (it is a $+$) and use that shortcut with $a =$ square root of 1st term and $b =$ square root of 2nd term

$$x^2 + 10x + 25$$
$$(x)^2 + 10x + (5)^2$$
$$(x)^2 + 2(x)(5) + (5)^2$$
$$(x + 5)(x + 5)$$
$$(x + 5)^2$$

It is OK to do these steps in your head

$$(\boldsymbol{x} + \mathbf{5})^2$$

Example 5

Factor $36x^2 - 84x + 49$

Step 1- Are the 1st and 3rd terms both perfect squares?

Yes – The square root of $36x^2$ is $6x$ and the square root of 49 is 7.

Step 2- Does the 2nd term = 2 • square root of 1st term • square root of 2nd term?

Yes – 2 • $6x$ • 7 = $84x$, which is the 2nd term.

Step 3- Look at the first operator (it is a $-$) and use that shortcut with $a =$ square root of 1st term and $b =$ square root of 2nd term

$$36x^2 - 84x + 49$$
$$(6x)^2 - 84x + (7)^2$$
$$(6x)^2 - 2(6x)(7) + (7)^2$$
$$(6x - 7)(6x - 7)$$
$$(6x - 7)^2$$

It is OK to do these steps in your head

$$\boldsymbol{(6x - 7)^2}$$