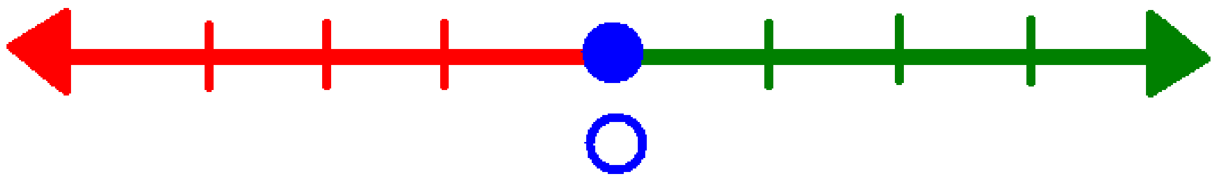

If we square a negative number, the result will always be a <u>positive</u> number.

$$(-1)(-1) = (-1)^2 = 1$$

If we square zero, the result will always be <u>zero</u>.

$$(0)(0) = \quad 0^2 \quad = 0$$

If we square a positive number, the result will always be a <u>positive</u> number.

$$(1)(1) = \quad 1^2 \quad = 1$$

That covers all the possibilities on the real number line. There aren't any *real numbers* we can square and get a result that is a <u>negative</u> number.

But we can make up a number… an *imaginary number* (it is not real)… that we can square and get a result that **is** a <u>negative</u> number. We will call this number $\boldsymbol{i}$.

Properties of the imaginary number $\boldsymbol{i}$		
$\boldsymbol{i = \sqrt{-1}}$	so	$\boldsymbol{i^2 = -1}$

Any time we see $\boldsymbol{i^2}$ we can substitute $-\mathbf{1}$.

Let's see this in action…

Properties of the imaginary number $\boldsymbol{i}$

$$\boldsymbol{i} = \sqrt{-1} \quad \text{so} \quad \boldsymbol{i}^2 = -1$$

<u>Example 1</u> Simplify $\sqrt{-100}$

$$\sqrt{-100}$$
$$\sqrt{100} \bullet \sqrt{-1}$$
$$10 \bullet \boldsymbol{i}$$
$$\boxed{\mathbf{10}\boldsymbol{i}}$$

<u>Example 2</u> Simplify $2\sqrt{-125}$

$$2\sqrt{-125}$$
$$2\sqrt{125} \bullet \sqrt{-1}$$
$$2(5\sqrt{5}) \bullet \boldsymbol{i}$$
$$\boxed{\mathbf{10}\boldsymbol{i}\sqrt{\mathbf{5}}}$$

[It's easier to read the *i* in front of the radical – it won't accidentally appear to be under the radical]

<u>Example 3</u> Simplify $6i \bullet 7i$

$$6i \bullet 7i$$
$$6 \bullet 7 \bullet i \bullet i$$
$$42\boldsymbol{i}^2$$
$$42(-1)$$
$$\boxed{\mathbf{-42}}$$

Properties of the imaginary number $\boldsymbol{i}$		
$\boldsymbol{i} = \sqrt{-1}$	so	$\boldsymbol{i}^2 = -1$

Example 4 Simplify $\sqrt{-2} \bullet \sqrt{-8}$

$$\sqrt{-2} \bullet \sqrt{-8}$$
$$\sqrt{2} \bullet \sqrt{-1} \bullet \sqrt{8} \bullet \sqrt{-1}$$
$$\sqrt{2} \bullet \boldsymbol{i} \bullet \sqrt{8} \bullet \boldsymbol{i}$$
$$\sqrt{16} \bullet \boldsymbol{i}^2$$
$$4(-1)$$
$$\boxed{\mathbf{-4}}$$

Example 5 Simplify $(-2i\sqrt{11})^2$

$$\left(-2i\sqrt{11}\right)^2$$
$$(-2)^2 \bullet \boldsymbol{i}^2 \bullet (\sqrt{11})^2$$
$$4 \bullet (-1) \bullet 11$$
$$\boxed{\mathbf{-44}}$$

Example 6 Simplify $\frac{7}{2i}$ [i is a radical, so it can't remain in the denominator]

$$\frac{7}{2i} \bullet \frac{i}{i} = \frac{7i}{2\boldsymbol{i}^2} = \frac{7i}{2(-1)} = \frac{7i}{-2}$$
$$\boxed{-\frac{\mathbf{7}\boldsymbol{i}}{\mathbf{2}}}$$

Properties of the imaginary number $\boldsymbol{i}$

$$\boldsymbol{i} = \sqrt{-1} \quad \text{so} \quad \boldsymbol{i^2 = -1}$$

<u>Example 7</u> Simplify $-6i(4-5i)$

$$-6i(4-5i)$$
$$-24i + 30\boldsymbol{i^2}$$
$$-24i + 30(\boldsymbol{-1})$$
$$-24i + -30$$

$$\boxed{\boldsymbol{-30 - 24i}}$$

<u>Example 8</u> Simplify $(3+7i)(2-3i)$

$$(3+7i)(2-3i)$$ [Use FOIL]
$$6 - 9i + 14i - 21\boldsymbol{i^2}$$
$$6 + 5i - 21(\boldsymbol{-1})$$
$$6 + 5i + 21$$

$$\boxed{\boldsymbol{27 + 5i}}$$

<u>Example 9</u> Simplify $(2-4i)^2$

$$(2-4i)^2$$
$$(2-4i)(2-4i)$$
$$4 - 8i - 8i + 16\boldsymbol{i^2}$$
$$4 - 16i + 16(\boldsymbol{-1})$$
$$4 - 16i - 16$$

$$\boxed{\boldsymbol{-12 - 16i}}$$