

A radical equation features a variable in a radicand (or in the “radical house”). Follow these steps to solve an equation with one radical:

Step 1- Get the radical term by itself on one side of the equation

Step 2- Square or cube (or whatever it takes) both sides of the equation so the radical will disappear

Step 3- Solve the resulting equation

Step 4- Check the solution(s) to make sure they really work. You may end up with extraneous solutions that you need to toss out.

Example 1 – Solve $\sqrt[3]{3x + 7} + 6 = 4$

Step 1- Get radical term by itself

$$\begin{aligned}\sqrt[3]{3x + 7} + 6 &= 4 \\ \sqrt[3]{3x + 7} &= -2\end{aligned}$$

Step 2- Cube (because the radical is a cube root) both sides

$$\begin{aligned}\sqrt[3]{3x + 7} &= -2 \\ (\sqrt[3]{3x + 7})^3 &= (-2)^3 \\ 3x + 7 &= -8\end{aligned}$$

Step 3- Solve the resulting equation

$$\begin{aligned}3x + 7 &= -8 \\ 3x &= -15 \\ x &= -5\end{aligned}$$

Step 4- Check solution

$$\begin{aligned}\sqrt[3]{3(-5) + 7} + 6 &= 4 \\ \sqrt[3]{-15 + 7} + 6 &= 4 \\ \sqrt[3]{-8} + 6 &= 4 \\ -2 + 6 &= 4 \\ 4 &= 4\end{aligned}$$



$$x = -5$$

Example 2 – Solve $x = 4 + 3\sqrt{x}$

Step 1- Get radical term by itself

$$x = 4 + 3\sqrt{x}$$

$$x - 4 = 3\sqrt{x}$$

Don't worry about the 3 in front of \sqrt{x} - since it is being multiplied, we're going to be OK when we square both sides.

Step 2- Square (because the radical is a square root) both sides

$$x - 4 = 3\sqrt{x}$$

$$(x - 4)^2 = (3\sqrt{x})^2$$

$$x^2 - 8x + 16 = 9x$$

Step 3- Solve the resulting equation

$$x^2 - 8x + 16 = 9x$$

$$x^2 - 17x + 16 = 0$$

$$(x - 16)(x - 1) = 0$$

$$x = 16, 1$$

Step 4- Check solutions

Check $x = 16$

$$(16) = 4 + 3\sqrt{(16)}$$

$$16 = 4 + 3(4)$$

$$16 = 4 + 12$$

$$16 = 16$$



Check $x = 1$

$$(1) = 4 + 3\sqrt{(1)}$$

$$1 = 4 + 3(1)$$

$$1 = 4 + 3$$

$$1 = 7$$



$$x = 16$$

Example 3 – Solve $x\sqrt{2} + 5 = 17$

This is a trick! This is NOT a radical equation because there are no variables in the radicand (the “radical house”). Solve this like any other equation you’ve been solving for years (just get x by itself).

$$x\sqrt{2} + 5 = 17$$

$$x\sqrt{2} = 12$$

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{12}{\sqrt{2}}$$

$$x = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$$

$$x = 6\sqrt{2}$$