

The change-of-base formula is often used to change a logarithm's base to 10. Since scientific calculators do logarithms with a base of 10, this is very useful!

Here's how it works (mathematically) – again, our focus is with a base of 10.

$$\log_u v = \frac{\log v}{\log u}$$

We could actually make our new logarithm become any base – for now our focus is with a base of 10 (that's what's available in a calculator).

Example 1

Find $\log_3 24$ to three decimal places.

We know the answer must be between 2 and 3 because $\log_3 9 = 2$ and $\log_3 27 = 3$. But that's all we can figure out right now. Let's use the change-of-base formula to do the work for us!

Step 1 – Use the change-of-base formula to re-write this as a fraction with two logarithms.

$$\log_3 24 = \frac{\log 24}{\log 3}$$

Step 2 – Use a scientific calculator to get the answer.

$$\frac{\log 24}{\log 3} = \frac{1.38021}{0.47712} \approx 2.893$$

Example 2

Find $\log_{12} 72$ to three decimal places.

Step 1 – Use the change-of-base formula .

$$\log_{12} 72 = \frac{\log 72}{\log 12}$$

Step 2 – Use a scientific calculator to get the answer.

$$\frac{\log 72}{\log 12} = \frac{1.85733}{1.07918} \approx 1.721$$