

Let's look at the **product property** of logarithms.

$$\log_b(u \cdot v) = \log_b u + \log_b v$$

This is not a surprise since we add exponents when we multiply with the same base!

Example 1- Evaluate $\log_3(3 \cdot 9)$

$$\log_3(3 \cdot 9) = \log_3 3 + \log_3 9 = 1 + 2 = \boxed{3}$$

$\log_3 3 = 1$ $\log_3 9 = 2$

Example 2- Evaluate $\log_8 64x$

$$\log_8 64x = \log_8 64 + \log_8 x = \boxed{2 + \log_8 x}$$

$\log_8 64 = 2$

Let's look at the **quotient property** of logarithms.

$$\log_b \left(\frac{u}{v} \right) = \log_b u - \log_b v$$

This makes sense since we subtract exponents when we divide with the same base!

Example 3- Evaluate $\log_5 \left(\frac{5}{125} \right)$

$$\log_5 \left(\frac{5}{125} \right) = \log_5 5 - \log_5 125 = 1 - 3 = \boxed{-2}$$

$\log_5 5 = 1$

$\log_5 125 = 3$

Let's look at the **power property** of logarithms

$$\log_b u^n = n \log_b u$$

Example 4- Evaluate $\log_3 25$

$$\log_3 25 = \log_3 5^2 = \mathbf{2} \log_3 5$$

Example 5- Evaluate $\log_4 64^3$

$$\log_4 64^3 = \mathbf{3} \log_4 64 = 3(3) = \boxed{9}$$

$\log_4 64 = 3$

Let's put these properties together!

Example 6 - Evaluate $\log \frac{7w^3}{m^4}$

We have multiplication and division both happening. Multiplication makes us add; division makes us subtract.

$$\log \frac{7w^3}{m^4} = \log 7 + \log w^3 - \log m^4$$

Next, let's take care of those exponents (use the power property).

$$\log 7 + \log w^3 - \log m^4 = \boxed{\log 7 + 3 \log w - 4 \log m}$$

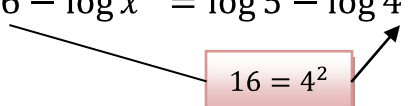
Example 7 - Evaluate $\log \frac{5}{16x^6}$

We have multiplication and division both happening. Multiplication makes us add; division makes us subtract.

$$\log \frac{5}{16x^6} = \log 5 - \log 16 - \log x^6$$

Next, let's take care of those exponents (use the power property).

$$\log 5 - \log 16 - \log x^6 = \log 5 - \log 4^2 - \log x^6 = \boxed{\log 5 - 2 \log 4 - 6 \log x}$$



$$16 = 4^2$$