Natural logarithms have a base of e. There is a special way to write natural logarithms.

$\log_e$  $= \ln$ 

Whenever you see $\ln$ , you know the base is e. This means we can rewrite $\ln$  as an exponential expression like this…

$\ln$  $= x \longrightarrow \log_e$  $= x \longrightarrow e^x =$ 

You eventually would like to be able to do the middle step in your head, but it is OK to write it out for now.

Example 1- Write $\ln 22 = 3.091$ in exponential form.

$$\ln 22 = 3.091 \longrightarrow \log_e 22 = 3.091 \longrightarrow \boxed{\boldsymbol{e^{3.091} = 22}}$$

Example 2- Write $e^4 = 54.598$ in logarithmic form.

$$e^4 = 54.598 \longrightarrow \log_e 54.598 = 4 \longrightarrow \boxed{\mathbf{\ln 54.598 = 4}}$$

We love $\mathbf{\ln e}$ because $\mathbf{\ln e = 1}$! Check it out...

$$\ln e = x \longrightarrow \log_e e = x \longrightarrow e^x = e \longrightarrow e^x = e^1 \longrightarrow x = 1$$

$$\boxed{\ln e = 1}$$

All our previous logarithm rules also apply to $\ln$ because $\ln$ is a logarithm!

Example 3- Simplify $\ln e^5$.

We can start by moving the 5 from the exponent spot to the front.

$$\ln e^5 = 5 \ \ln e$$

And we know $\ln e = 1$, so...

$$5 \ \ln e = 5(1) = \boxed{\mathbf{5}}$$

Example 4- Write $\ln 20 - \ln 4$ as a single logarithm

Since we are subtracting logarithms, we must be dividing (use the same logarithm rules we've already learned).

$$\ln 20 - \ln 4 \longrightarrow \ln \frac{20}{4} \longrightarrow \boxed{\mathbf{\ln 5}}$$

<u>Example 5</u>- Solve the following equation for x (leave answer in terms of e).

$$\ln x^6 = 42$$

We can start by moving the 6 from the exponent spot to the front.

$$\ln x^6 = 42$$
$$6 \ln x = 42$$

Now we can divide both sides by 6 because we want $\mathbf{\ln x}$ by itself on one side of the equation.

$$6 \ln x = 42$$
$$\ln x = 7$$

Let's rewrite $\mathbf{\ln x}$ as $\mathbf{\log_e x}$.

$$\ln x = 7$$
$$\log_e x = 7$$

We can change this from logarithmic form to exponential form, and we will have solved for x!

$$\log_e x = 7$$

$$e^7 = x$$

$$\boxed{\boldsymbol{x = e^7}}$$

<u>Example 6</u>- Solve the following equation for x (leave answer in terms of natural logarithms).

$$e^x = 9$$

We have already learned that when the variable is in the exponent spot and we can't easily get the same bases on both sides of an equation, we are going to need to use logarithms on both sides.

Because the number e is involved, we will use $\ln$ on both sides.
If the number e was not present, we would use $\log$ on both sides (as we have already done in this unit).

$$e^x = 9$$
$$\ln e^x = \ln 9$$

We can move the x from the exponent spot to the front.

$$\ln e^x = \ln 9$$
$$x \ln e = \ln 9$$

We know $\ln e = 1$.

$$x \ln e = \ln 9$$
$$x(1) = \ln 9$$

$$\boxed{\boldsymbol{x = \ln 9}}$$