The most important part of adding (or subtracting) rational expressions is to <u>make</u> sure all the expressions have a **common denominator**.

Example 1 – Simplify
$$\frac{7}{4x} + \frac{9}{4x}$$

Since the denominators are already the same, we can make a single fraction by adding the numerators and keeping the common denominator.

$$\frac{7}{4x} + \frac{9}{4x} = \frac{7+9}{4x} = \frac{16}{4x}$$

The last step is to see if there are any common factors that can be reduced in the numerator and denominator.

$$\frac{16}{4x} = \frac{4(4)}{4x} = \frac{4}{x}$$

$$\boxed{\frac{4}{x}}$$

Example 2 – Simplify
$$\frac{5}{3m} + \frac{5}{4m}$$

Step 1 – Factor the denominators

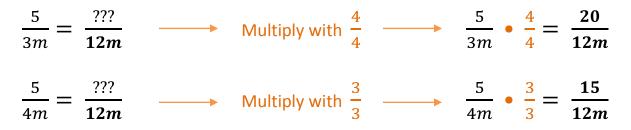
Our denominators cannot be factored.

Step 2 – Determine a common denominator to use (ignore the numerators)

Our denominators are 3m and 4m. The best (and least) common denominator to use is

$$3 \cdot 4 \cdot m = 12m$$

Step 3 – Rewrite each fraction with the common denominator from Step 2



Step 4 – Rewrite the problem with the new fractions

$$\frac{20}{12m} + \frac{15}{12m}$$

Step 5 – Add the fractions to make a single fraction (add the numerators and leave the denominators alone)

$$\frac{20}{12m} + \frac{15}{12m} = \frac{20 + 15}{12m} = \frac{35}{12m}$$

Step 6 – If possible, simplify the fraction (this may require factoring the numerator)

Our answer cannot be simplified.

35	
12 <i>m</i>	,

Example 3 – Simplify
$$\frac{3}{w^2-6w} + \frac{6}{w^2-3w-18}$$

Step 1 – Factor the denominators

Our denominators are $w^2 - 6w$ and $w^2 - 3w - 18$. Let's factor them.

$$w^{2} - 6w = w(w - 6) \qquad \qquad w^{2} - 3w - 18 = (w - 6)(w + 3)$$
$$\frac{3}{w(w - 6)} + \frac{6}{(w - 6)(w + 3)}$$

Step 2– Determine a common denominator to use (ignore the numerators).

The best (and least) common denominator to use is

$$w \cdot (w - 6) \cdot (w + 3) = w(w - 6)(w + 3)$$

Step 3 – Rewrite each fraction with the common denominator from Step 2

$$\frac{6}{(w-6)(w+3)} = \frac{???}{w(w-6)(w+3)} \qquad \qquad \text{Multiply with } \frac{w}{w}$$
$$\frac{6}{(w-6)(w+3)} \bullet \frac{w}{w} = \frac{6w}{w(w-6)(w+3)} \bullet$$

Step 4 – Rewrite the problem with the new fractions

$$\frac{3w+9}{w(w-6)(w+3)} + \frac{6w}{w(w-6)(w+3)}$$

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Step 5 – Add the fractions to make a single fraction (add the numerators and leave the denominators alone)

$$\frac{3w+9}{w(w-6)(w+3)} + \frac{6w}{w(w-6)(w+3)} = \frac{3w+9+6w}{w(w-6)(w+3)} = \frac{9w+9}{w(w-6)(w+3)}$$

Step 6 – If possible, simplify the fraction (this may require factoring the numerator)

We <u>can</u> factor the numerator.

$$\frac{9w+9}{w(w-6)(w+3)} = \frac{9(w+1)}{w(w-6)(w+3)}$$

However, there aren't any common factors to reduce.

$$(\frac{9(w+1)}{w(w-6)(w+3)})$$
 or $(\frac{9w+9}{w(w-6)(w+3)})$

Example 4 – Simplify $\frac{x+1}{x^2+6x+9} - \frac{1}{x^2-9}$

Step 1 – Factor the denominators

Our denominators are $x^2 + 6x + 9$ and $x^2 - 9$. Let's factor them.

$$x^{2} + 6x + 9 = (x+3)(x+3) \text{ or } (x+3)^{2} \qquad x^{2} - 9 = (x+3)(x-3)$$
$$\frac{x+1}{(x+3)^{2}} - \frac{1}{(x+3)(x-3)}$$

Step 2– Determine a common denominator to use (ignore the numerators).

The best (and least) common denominator to use is $(x + 3)^2(x - 3)$

Step 3 – Rewrite each fraction with the common denominator from Step 2

Step 4 – Rewrite the problem with the new fractions

$$\frac{x^2 - 2x - 3}{(x+3)^2(x-3)} - \frac{x+3}{(x+3)^2(x-3)}$$

Step 5 – Subtract the fractions to make a single fraction (subtract the numerators and leave the denominators alone)

$$\frac{x^2 - 2x - 3}{(x+3)^2(x-3)} - \frac{x+3}{(x+3)^2(x-3)} =$$

$$\frac{x^2 - 2x - 3 - (x+3)}{(x+3)^2(x-3)} =$$

$$\frac{x^2 - 2x - 3 - x - 3}{(x+3)^2(x-3)} =$$

$$\frac{x^2 - 3x - 6}{(x+3)^2(x-3)}$$

Step 6 – If possible, simplify the fraction (this may require factoring the numerator)

We can't factor the numerator, so we are done.

$$\left(\frac{x^2 - 3x - 6}{(x+3)^2(x-3)}\right)$$