The most important part of adding (or subtracting) rational expressions is to <u>make sure all the expressions have a **common denominator**</u>.

<u>Example 1</u> – Simplify $\frac{7}{4x}+\frac{9}{4x}$

Since the denominators are already the same, we can make a single fraction by adding the numerators and keeping the common denominator.

$$\frac{7}{4x}+\frac{9}{4x}=\frac{7+9}{4x}=\frac{16}{4x}$$

The last step is to see if there are any common factors that can be reduced in the numerator and denominator.

$$\frac{16}{4x}=\frac{4(4)}{4x}=\frac{4}{x}$$

$$\boxed{\frac{4}{x}}$$

Example 2 – Simplify $\frac{5}{3m} + \frac{5}{4m}$

Step 1 – Factor the denominators

Our denominators cannot be factored.

Step 2 – Determine a common denominator to use (ignore the numerators)
Our denominators are $3m$ and $4m$. The best (and least) common denominator to use is

$$3 \bullet 4 \bullet m = \mathbf{12}\boldsymbol{m}$$

Step 3 – Rewrite each fraction with the common denominator from Step 2

$\frac{5}{3m} = \frac{???}{\mathbf{12}\boldsymbol{m}}$ ⟶ Multiply with $\frac{4}{4}$ ⟶ $\frac{5}{3m} \bullet \frac{4}{4} = \frac{\mathbf{20}}{\mathbf{12}\boldsymbol{m}}$

$\frac{5}{4m} = \frac{???}{\mathbf{12}\boldsymbol{m}}$ ⟶ Multiply with $\frac{3}{3}$ ⟶ $\frac{5}{4m} \bullet \frac{3}{3} = \frac{\mathbf{15}}{\mathbf{12}\boldsymbol{m}}$

Step 4 – Rewrite the problem with the new fractions

$$\frac{20}{12m} + \frac{15}{12m}$$

Step 5 – Add the fractions to make a single fraction (add the numerators and leave the denominators alone)

$$\frac{20}{12m} + \frac{15}{12m} = \frac{20 + 15}{12m} = \frac{35}{12m}$$

Step 6 – If possible, simplify the fraction (this may require factoring the numerator)

Our answer cannot be simplified. $\boxed{\frac{\mathbf{35}}{\mathbf{12}\boldsymbol{m}}}$

<u>Example 3</u> – Simplify $\frac{3}{w^2-6w}+\frac{6}{w^2-3w-18}$

Step 1 – Factor the denominators

Our denominators are w^2-6w and $w^2-3w-18$. Let's factor them.

$w^2-6w=w(w-6)$ $\qquad$ $w^2-3w-18=(w-6)(w+3)$

$$\frac{3}{w(w-6)}+\frac{6}{(w-6)(w+3)}$$

Step 2– Determine a common denominator to use (ignore the numerators).

The best (and least) common denominator to use is
$w \bullet (w-6) \bullet (w+3) = \mathbf{w(w-6)(w+3)}$

Step 3 – Rewrite each fraction with the common denominator from Step 2

$\frac{3}{w(w-6)}=\frac{???}{\mathbf{w(w-6)(w+3)}}$ → Multiply with $\frac{w+3}{w+3}$

$\frac{3}{w(w-6)} \bullet \frac{w+3}{w+3}=\frac{\mathbf{3w+9}}{\mathbf{w(w-6)(w+3)}}$

$\frac{6}{(w-6)(w+3)}=\frac{???}{\mathbf{w(w-6)(w+3)}}$ → Multiply with $\frac{w}{w}$

$\frac{6}{(w-6)(w+3)} \bullet \frac{w}{w}=\frac{\mathbf{6w}}{\mathbf{w(w-6)(w+3)}}$

Step 4 – Rewrite the problem with the new fractions

$$\frac{3w+9}{w(w-6)(w+3)}+\frac{6w}{w(w-6)(w+3)}$$

Step 5 – Add the fractions to make a single fraction (add the numerators and leave the denominators alone)

$$\frac{3w+9}{w(w-6)(w+3)}+\frac{6w}{w(w-6)(w+3)}=\frac{3w+9+6w}{w(w-6)(w+3)}=\frac{9w+9}{w(w-6)(w+3)}$$

Step 6 – If possible, simplify the fraction (this may require factoring the numerator)

We <u>can</u> factor the numerator.

$$\frac{9w+9}{w(w-6)(w+3)}=\frac{9(w+1)}{w(w-6)(w+3)}$$

However, there aren't any common factors to reduce.

$$\boxed{\mathbf{\frac{9(w+1)}{w(w-6)(w+3)}}} \quad \text{or} \quad \boxed{\mathbf{\frac{9w+9}{w(w-6)(w+3)}}}$$

Example 4 – Simplify $\frac{x+1}{x^2+6x+9} - \frac{1}{x^2-9}$

Step 1 – Factor the denominators

Our denominators are x^2+6x+9 and x^2-9. Let's factor them.

$x^2+6x+9 = (x+3)(x+3)$ *or* $(x+3)^2$ $\qquad x^2-9 = (x+3)(x-3)$

$$\frac{x+1}{(x+3)^2} - \frac{1}{(x+3)(x-3)}$$

Step 2– Determine a common denominator to use (ignore the numerators).

The best (and least) common denominator to use is

$$\mathbf{(x+3)^2(x-3)}$$

Step 3 – Rewrite each fraction with the common denominator from Step 2

$\frac{x+1}{(x+3)^2} = \frac{???}{(x+3)^2(x-3)}$ → Multiply with $\frac{x-3}{x-3}$

$\frac{x+1}{(x+3)^2} \bullet \frac{x-3}{x-3} = \frac{x^2-2x-3}{(x+3)^2(x-3)}$ ← [use FOIL]

$\frac{1}{(x+3)(x-3)} = \frac{???}{(x+3)^2(x-3)}$ → Multiply with $\frac{x+3}{x+3}$

$\frac{1}{(x+3)(x-3)} \bullet \frac{x+3}{x+3} = \frac{x+3}{(x+3)^2(x-3)}$ ←

Step 4 – Rewrite the problem with the new fractions

$$\frac{x^2-2x-3}{(x+3)^2(x-3)} - \frac{x+3}{(x+3)^2(x-3)}$$

Step 5 – Subtract the fractions to make a single fraction (subtract the numerators and leave the denominators alone)

$$\frac{x^2-2x-3}{(x+3)^2(x-3)}-\frac{x+3}{(x+3)^2(x-3)}=$$

$$\frac{x^2-2x-3-(x+3)}{(x+3)^2(x-3)}=$$

Use parentheses ( ) when you subtract to help you accurately distribute the negative [–] sign

$$\frac{x^2-2x-3-x-3}{(x+3)^2(x-3)}=$$

$$\frac{x^2-3x-6}{(x+3)^2(x-3)}$$

Step 6 – If possible, simplify the fraction (this may require factoring the numerator)

We can't factor the numerator, so we are done.

$$\boxed{\boldsymbol{\frac{x^2-3x-6}{(x+3)^2(x-3)}}}$$