A *circle* is the set of all points (in a plane) the same distance (the *radius*) away from a fixed point (the *center*).

Do you notice the similarity between the equation of a circle and the Pythagorean Theorem ($a^2 + b^2 = c^2$) and the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 =}\ D$?

Equation of a Circle (Standard Form)
$(x - h)^2 + (y - k)^2 = r^2$
Center: (h, k) Radius: r

<u>Example 1</u>

Write the equation of a circle with center $(3, 2)$ and radius 10.

Step 1- Identify $h, k, and\ r$. Put these values in the equation of a circle.

$$h = 3 \quad k = 2 \qquad r = 10$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 3)^2 + (y - 2)^2 = 10^2$$

Step 2- Simplify the right side (leave the left side alone)

$$(x - 3)^2 + (y - 2)^2 = 10^2$$

$$\boxed{\boldsymbol{(x - 3)^2 + (y - 2)^2 = 100}}$$

Example 2

Write the equation of a circle with center $(-4, 0)$ and radius $2\sqrt{3}$.

Step 1- Identify $h, k, and\ r$. Put these values in the equation of a circle.

$$h = -4 \quad k = 0 \qquad r = 2\sqrt{3}$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (-4))^2 + (y - 0)^2 = (2\sqrt{3})^2$$

$$(x + 4)^2 + y^2 = (2\sqrt{3})^2$$

Step 2- Simplify the right side (leave the left side alone)

$$(x + 4)^2 + y^2 = (2\sqrt{3})^2$$

$$\boxed{\boldsymbol{(x + 4)^2 + y^2 = 12}}$$

Example 3

Graph the equation $x^2 + (y-3)^2 = 25$

Step 1- Identify $h, k, and\ r$.

$$h = 0 \quad k = 3 \qquad\qquad r = \sqrt{25} = 5$$

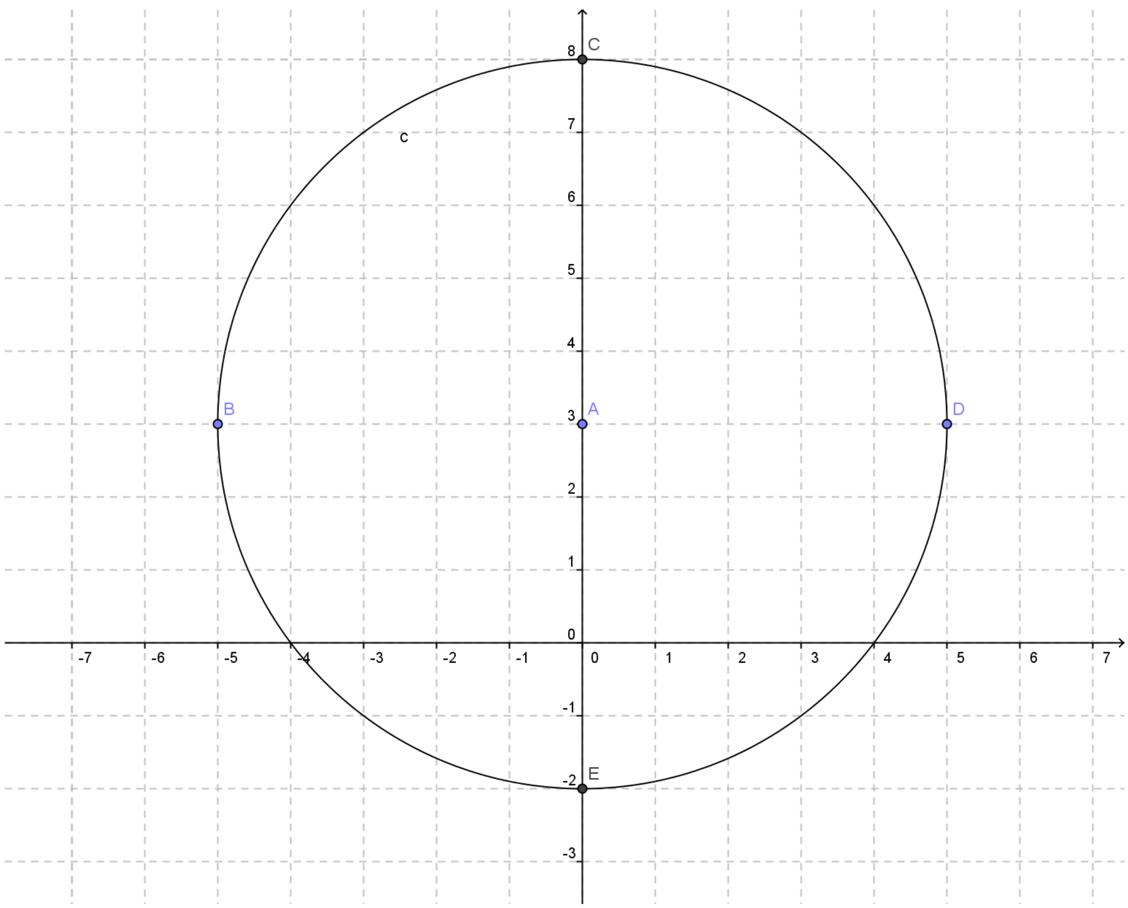
Step 2- Put the center (h, k) on the graph

The first point we put on the graph is $(0, 3)$.

Step 3- Use the radius to make four points on the circle. Go up, down, left, and right the distance of the radius to find the points.

This is how the four points – $(5, 3), (-5, 3), (0, -2), \& (0, 8)$ – were placed on the graph. Each point is 5 units away from the center.

Step 4- Sketch a circle using those four points.



Example 4

If the graph of the following equation is a circle, find its center and radius. If it isn't a circle, say so.

$$x^2 + y^2 + 8x - 2y - 19 = 0$$

Step 1- Make sure the coefficients of x^2 & y^2 are the same. If they aren't, this isn't a circle.

The coefficients of both x^2 & y^2 are 1, so this could be a circle.

Step 2- Move the "free number" to the right side and rearrange the x^2 & x terms to be together and the y^2 & y terms to be together. Leave some space for Step 3.

$$x^2 + y^2 + 8x - 2y - 19 = 0$$
$$x^2 + y^2 + 8x - 2y = 19$$
$$(x^2 + 8x + \quad) + (y^2 - 2y + \quad) = 19$$

Step 3- Complete the square two times – once for the x-group and once for the y-group.

$$(x^2 + 8x + \square\ ) + (y^2 - 2y + \square\ ) = 19 + \square + \square$$

For the x-group: $8 \div 2 = 4$ $(4)^2 = \mathbf{16}$

For the y-group: $-2 \div 2 = -1$ $(-1)^2 = \mathbf{1}$

$$(x^2 + 8x + 16\ ) + (y^2 - 2y + 1\ ) = 19 + 16 + 1$$

$$\boldsymbol{(x+4)^2 + (y-1)^2 = 36}$$

Step 4- Make sure the number on the right side is positive. This number equals $(\text{radius})^2$, so it can't be negative (that would make the distance imaginary) and it can't be zero (there would be no distance at all).

The number on the right side is 36, and $36 > 0$, so we are OK. We have found the equation of the circle in standard form.

Step 5- Identify $h, k, and\ r$.

$h = -4$ $k = 1$ $r = \sqrt{36} = 6$

$$\boxed{\textbf{Center: } (-4, 1) \quad \textbf{Radius: } 6}$$