A sequence is an **arithmetic sequence** if the difference between consecutive terms is always the same.

Example 1- Is the following sequence arithmetic?

$$8, 11, 14, 17, \ldots$$

The difference (change) between consecutive terms is always the same (it is always $+3$), so **YES**, it is an arithmetic sequence.

Example 2- Is the following sequence arithmetic?

$$1, 3, 6, 10, 15, \ldots$$

The difference (change) between consecutive terms is not always the same, so **NO**, it is not arithmetic sequence.

Example 3- Is the following sequence arithmetic?

$$17, 11, 5, -1, -7, \ldots$$

The difference (change) between consecutive terms is always the same (it is always -6), so **YES**, it is an arithmetic sequence.

We can make a rule for an arithmetic sequence as long as we know…

1) the difference (change) between consecutive terms (it is always the same);
2) the value of the 1st term.

We will call the difference d and we will call the 1st term a_1.

The rule for any nth term (it could be the 9th, 15th, 71st, etc.) of an arithmetic sequence is

$$a_n = a_1 + d(n - 1)$$

1st term

Difference

Rule for an arithmetic sequence

Example 4- Find the rule for the following arithmetic sequence:

$$8, 11, 14, 17, \ldots$$

The difference $d = +3$ and the 1st term $a_1 = 8$.

$$a_n = a_1 + d(n - 1)$$
$$a_n = 8 + 3(n - 1)$$
$$a_n = 8 + 3n - 3$$

$$\boxed{\boldsymbol{a_n = 3n + 5}}$$

Example 5- Find the 32nd term of the sequence in Example 4.

It would take a long time to write out the first 32 terms. It is quicker to use the rule we just found. Since we want the 32nd term, we will make $n = 32$.

$$a_n = 3n + 5$$
$$a_{32} = 3(32) + 5 = 96 + 5 = 101$$

$$\boxed{\boldsymbol{a_{32} = 101}}$$

Example 6- Find the 15th term of the following arithmetic sequence:

$$17, 11, 5, -1, -7, \ldots$$

We were not told to find a rule for this sequence, but that is what we must do first. After we find a rule, we can use that rule to discover the 15th term.

Step 1- Find the rule (Need the difference and the 1st term)
The difference $d = -6$ and the 1st term $a_1 = 17$.

$$a_n = a_1 + d(n - 1)$$
$$a_n = 17 + (-6)(n - 1)$$
$$a_n = 17 - 6n + 6$$

$$a_n = 23 - 6n \;\; (\text{or } a_n = -6n + 23)$$

Step 2- Find the 15th term
We will make $n = 15$

$$a_n = 23 - 6n$$
$$a_{15} = 23 - 6(15) = 23 - 90 = -67$$

$$\boxed{\boldsymbol{a_{15} = -67}}$$

The sum of the first n terms of an arithmetic series is…

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Sum of a finite arithmetic series

In other words, it is (the # of terms) • (average of the 1st term and last term)

Example 7- Find the sum of the first 28 terms of the for the following arithmetic series:

$$9 + 19 + 29 + 39 + 49 + \cdots$$

We need to know the # of terms, the 1st term, and the last (or 28th) term.

the # of terms $= 28$ 1st term $= 9$ last (or 28th) term$= ???$

We will have to find a rule to get the 28th term.
The 1st term $= 9$ and the difference $= +10$

$$a_n = 9 + 10(n-1)$$
$$a_n = 9 + 10n - 10$$
$$a_n = 10n - 1$$
so
$$a_{28} = 10(28) - 1 = 280 - 1 = 279$$

Now we can find the sum of the first 28 terms.

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$
$$S_{28} = 28\left(\frac{9 + 279}{2}\right)$$
$$S_{28} = 28\left(\frac{288}{2}\right) = 28(144) = 4032$$

$$\boldsymbol{S_{28} = 4032}$$

<u>Example 8</u>- Find the sum of the series $\sum_{n=1}^{25}(3n-2)$

We need to know the # of terms, the 1st term, and the last (or 25th) term.

We already have a rule ready for us – it is $3n-2$.

the # of terms $= 25$
1st term $= 3(1) - 2 = 3 - 2 = 1$
last (or 25th) term$= \ 3(25) - 2 = 75 - 2 = 73$

Now we can find the sum of the first 25 terms.

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$
$$S_{25} = 25\left(\frac{1 + 73}{2}\right)$$
$$S_{25} = 25\left(\frac{74}{2}\right) = 25(37) = 925$$

$$\boxed{\boldsymbol{S_{25} = 925}}$$