A sequence is a **geometric sequence** if the ratio (the number by which you multiply) between consecutive terms is always the same.

Example 1- What is the ratio of the following geometric sequence?

$$2, 6, 18, 54, 162, \ldots$$

We are multiplying by $+3$ every time ($\frac{6}{2} = +3$; $\frac{18}{6} = +3$; etc.)

Example 2- What is the ratio of the following geometric sequence?

$$16, 4, 1, \frac{1}{4}, \frac{1}{16}, \ldots$$

We are multiplying by $+\frac{1}{4}$ every time ($\frac{4}{16} = +\frac{1}{4}$; $\frac{1}{4} = +\frac{1}{4}$; etc.)

Example 3- What is the ratio of the following geometric sequence?

$$7, -14, 28, -56, 112, -224, \ldots$$

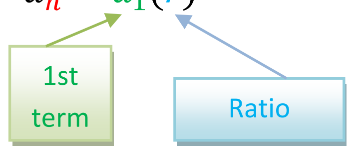
We are multiplying by -2 every time ($\frac{-14}{7} = -2$; $\frac{-14}{7} = -2$; etc.)

We can make a rule for a geometric sequence as long as we know…

1) the ratio (amount by which we multiply) between consecutive terms (it is always the same);
2) the value of the 1st term.

We will call the ratio r and we will call the 1st term a_1.

$$a_n = a_1(r)^{n-1}$$



Rule for a geometric sequence

Example 4 – Find the rule for the following geometric sequence?

$$2, 6, 18, 54, 162, \ldots$$

The ratio $r = +3$ and the 1st term $a_1 = 2$.

$$a_n = a_1(r)^{n-1}$$

$$\boldsymbol{a_n = 2(3)^{n-1}}$$

Example 5– Find the rule for the following geometric sequence?

$$7, -14, 28, -56, 112, -224, \ldots$$

The ratio $r = -2$ and the 1st term $a_1 = 7$.

$$a_n = a_1(r)^{n-1}$$

$$\boldsymbol{a_n = 7(-2)^{n-1}}$$

The sum of the first n terms of a finite geometric series is…

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Sum of a finite geometric series

a_1 is the 1st term, r is the ratio between consecutive terms, n is the # of terms.

Example 6- Find the sum of the first 10 terms of the for the following geometric series:

$$2, 6, 18, 54, 162, \ldots$$

The 1st term $a_1 = 2$, the ratio $r = 3$, and the number of terms $n = 10$.

$$S_{10} = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_{10} = 2\left(\frac{1-3^{10}}{1-3}\right)$$

$$S_{10} = 2\left(\frac{1-59{,}049}{1-3}\right)$$

$$S_{10} = 2\left(\frac{-59{,}048}{-2}\right)$$

$$\boldsymbol{S_{10} = 59{,}048}$$

This was a lot faster than trying to add

$$2 + 6 + 18 + 54 + 162 + 486 + 1458 + 4374 + 13{,}122 + 39{,}366$$