Example 1- There are 25 students in the classroom competing for ice cream prizes. First place gets a triple-scoop sundae, second place earns a two-scoop sundae, and third place receives a one-scoop sundae. How many ways can students win the prizes?

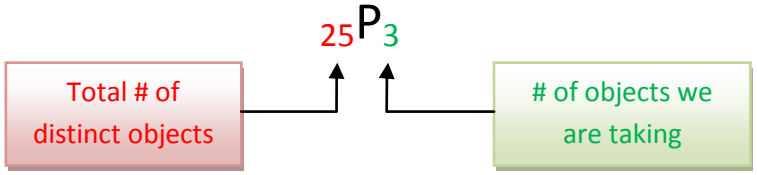
Notice that **the order of selection does matter** (if you are in first place, you get more scoops than third place!) and repeating is not going to be possible.

How many students could win first place?
How many students could win second place (knowing that the first place winner is not a factor anymore)?
How many students could win third place (first & second place winners won't come in third)?

There are **13,800** ways students could win these prizes

*** A shortcut!!! You can use the **nPr** button on your calculator to do the math on this problem. If you have a TI-30X II S, press the **PRB** button to see some choices (**nPr** is the first choice).

We have 25 students taken 3 at a time (remember, the order does matter- first place isn't the same as third place!). We can write this as…



If you enter **25**, followed by **nPr**, and then **3**, your calculator will give you **13,800**!

Factorial (!)

$$4! = (4)(3)(2)(1) = 24$$
$$8! = (8)(7)(6)(5)(4)(3)(2)(1) = 40{,}320$$

Example 2- Mr. Bower has 25 students and 25 desks. How many ways could the order of the seating be arranged?

Student 1- There are 25 desks available. Student 1 sits in one of the empty seats.
Student 2- There are now 24 desks available. Student 2 sits in one of the empty seats.
Student 3- There are now 23 desks available. Student 3 sits in one of the empty seats.
Student 4- There are now 22 desks available. Student 4 sits in one of the empty seats.
Student 5- There are now 21 desks available. Student 5 sits in one of the empty seats.

Student 23- There are now 3 desks available. Student 23 sits in one of the empty seats.
Student 24- There are now 2 desks available. Student 24 sits in one of the empty seats.
Student 25- There is now 1 desk available. Student 25 sits in one of the empty seats.

The answer to this problem is

$$\mathbf{25}! = (25)(24)(23)(22)(21)(20)(19)(18)(17)(16)(15)(14)(13)(12)(11)(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)$$

15,511,210,043,330,985,984,000,000

*** Tip!!! Look for the **x!** or **n!** on your calculator. If you have a TI-30X II S, press the **PRB** button to see some choices (**!** is the third choice).

Permutations w/repetition

Example 3- How many ways can the letters HONEY be distinguishably rearranged?

The answer is $5! = (5)(4)(3)(2)(1) = \mathbf{120}$

Example 4- How many ways can the letters PUPPY be distinguishably rearranged?

At first glance, you might think the answer to this problem is also 120. But some of the arrangements we get look alike.

PUPPY looks different from PUPPY because the letters have been colored.
PUPPY looks the same as PUPPY – it is impossible to tell if any of the P's have been moved around because they all look the same.

How many letters are in PUPPY? **5**
Which letters are repeated? How many times? **P → 3 times**

Instead of just using $5!$, we will divide out the repeating objects…

$$\frac{5!}{3!} = \frac{(5)(4)(3)(2)(1)}{(3)(2)(1)} = (5)(4) = \mathbf{20}$$

Example 5- How many ways can the letters MISSISSIPPI be distinguishably rearranged?

How many letters are in MISSISSIPPI? **11**
Which letters are repeated? How many times? **I → 4 times**
S → 4 times
P → 2 times

$$\frac{11!}{4! \bullet 4! \bullet 2!} = \frac{(11)(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(4)(3)(2)(1)(4)(3)(2)(1)(2)(1)} = \mathbf{34,650}$$