Let's see how we can solve quadratic equations ($ax^2 + bx + c = 0$) by **completing the square**.

Example 1
Solve $x^2 + 6x - 9 = 0$ by completing the square.

Step 1- Make the coefficient in front of $x^2 = 1$ (for $ax^2, a = 1$)
The coefficient of x^2 already is 1 in our problem, so we can skip this step.

Step 2- Rearrange the equation so $x^2 + bx$ is alone on the left side. Leave some room for a number to be added during Step 3.

$$x^2 + 6x - 9 = 0$$
$$x^2 + 6x = 9$$
$$x^2 + 6x + \blacksquare = 9 + \blacksquare$$

Step 3- Complete the square for the left side, and be sure to add this new number to BOTH SIDES to keep the equation balanced.

The left side is $x^2 + 6x + \blacksquare$

The coefficient of the middle term is 6 $\longrightarrow$ $6 \div 2 = 3$ $\longrightarrow$ $(3)^2 = 9$

We will add 9 to BOTH SIDES of the equation, and this will allow us to complete the square on the left side.

$$x^2 + 6x + \blacksquare = 9 + \blacksquare$$
$$x^2 + 6x + 9 = 9 + 9$$
Make a binomial square $\longrightarrow$ $(x + 3)^2 = 18$

Step 4- Take the square root of BOTH SIDES of the equation to solve for the variable.

$$\sqrt{(x + 3)^2} = \sqrt{18}$$
$$x + 3 = \pm 3\sqrt{2}$$

$$\boxed{\boldsymbol{x = -3 \pm 3\sqrt{2}}}$$

Example 2

Solve $5x^2 - 10x + 30 = 0$ by completing the square.

Step 1- Make the coefficient in front of $x^2 = 1$ (for $ax^2, a = 1$)

The coefficient of x^2 is 5 in our problem, so we will factor out 5 from each term.

$$5x^2 - 10x + 30 = 0$$
$$5(x^2 - 2x + 6) = 0$$
$$x^2 - 2x + 6 = 0$$

← Now divide by **5**

Step 2- Rearrange the equation so $x^2 + bx$ is alone on the left side. Leave some room for a number to be added during Step 3.

$$x^2 - 2x + 6 = 0$$
$$x^2 - 2x = -6$$
$$x^2 - 2x + \blacksquare = -6 + \blacksquare$$

Step 3- Complete the square for the left side, and be sure to add this new number to BOTH SIDES to keep the equation balanced.

The left side is $x^2 - 2x + \blacksquare$

The coefficient of the middle term is -2 ⟶ $-2 \div 2 = -1$ ⟶ $(-1)^2 = 1$

We will add 1 to BOTH SIDES of the equation, and this will allow us to complete the square on the left side.

$$x^2 + 6x + = -6 + \blacksquare$$
$$x^2 - 2x + 1 = -6 + 1$$
$$(x - 1)^2 = -5$$

Make a binomial square →

Step 4- Take the square root of BOTH SIDES of the equation to solve for the variable.

$$\sqrt{(x-1)^2} = \sqrt{-5}$$
$$x - 1 = \pm i\sqrt{5}$$

$$\boldsymbol{x = 1 \pm i\sqrt{5}}$$