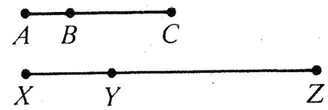
NAME ______________________ DATE ______________

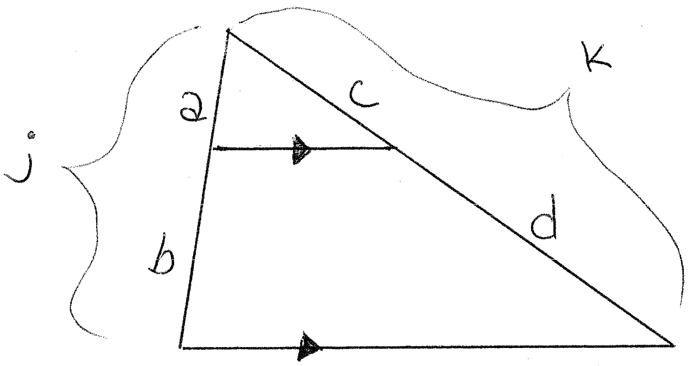
8-6 Proportional Lengths

Objectives: Apply the Triangle Proportionality Theorem and its corollary. Apply the Triangle Angle-Bisector Theorem.

If $\frac{AB}{BC} = \frac{XY}{YZ}$, then $\overline{AC}$ and $\overline{XZ}$ are said to be **divided proportionally.**



Triangle Proportionality Theorem	**If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.**



$\frac{a}{b} = \frac{c}{d}$ $\frac{a}{j} = \frac{c}{k}$ $\frac{b}{j} = \frac{d}{k}$

(of course, $j = a+b$ AND $k = c+d$)

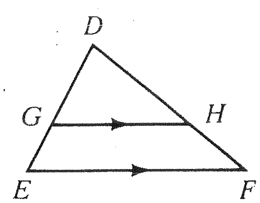
The properties of proportions allow the Triangle Proportionality Theorem to justify many equivalent proportions. Some of these equivalent proportions, along with informal statements describing the proportions, are given below.

$\frac{DG}{GE} = \frac{DH}{HF}$ $\frac{\text{larger piece}}{\text{smaller piece}} = \frac{\text{larger piece}}{\text{smaller piece}}$

$\frac{GE}{DE} = \frac{HF}{DF}$ $\frac{\text{smaller piece}}{\text{whole side}} = \frac{\text{smaller piece}}{\text{whole side}}$

$\frac{DG}{DE} = \frac{DH}{DF}$ $\frac{\text{larger piece}}{\text{whole side}} = \frac{\text{larger piece}}{\text{whole side}}$

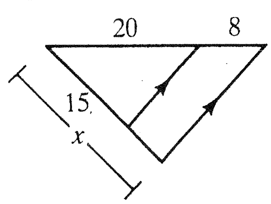
$\frac{GE}{HF} = \frac{DG}{DH} = \frac{DE}{DF}$ $\frac{\text{smaller piece}}{\text{smaller piece}} = \frac{\text{larger piece}}{\text{larger piece}} = \frac{\text{whole side}}{\text{whole side}}$



$\triangle DGH \sim \triangle DEF$, so some of these equivalent proportions could have been justified by similarity postulates or theorems. With so many equivalent proportions, most exercises can be done more than one way.

Example 1

Find the value of x.



Solution

$\frac{x}{15} = \frac{28}{20}$

$20x = 15(28)$

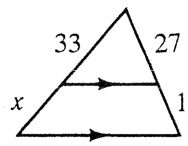
$x = 21$

$\left(\text{or } \frac{15}{x - 15} = \frac{20}{8}\right)$

Try these on your own or with a partner

Find the value of x.

1.



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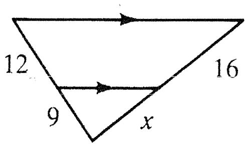

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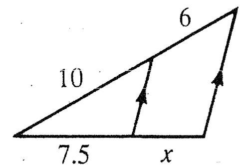

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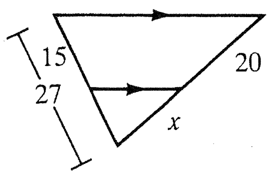

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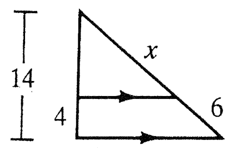

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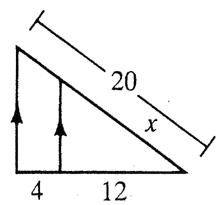

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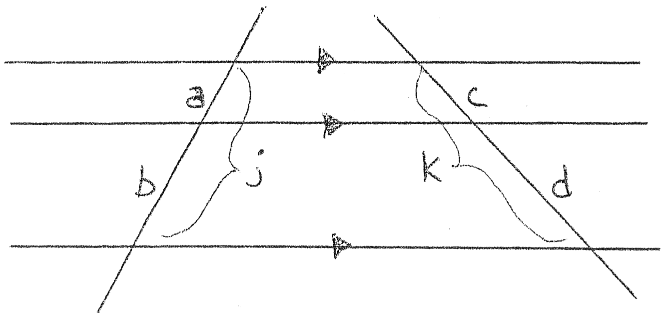
8–6 Proportional Lengths *(continued)*

You know: If three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. The following corollary to the Triangle Proportionality Theorem deals with parallel lines and proportional segments.

> **If three parallel lines intersect two transversals, then they divide the transversals proportionally.**

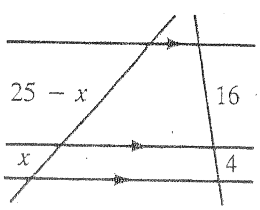


$$\frac{a}{b} = \frac{c}{d} \qquad \frac{a}{j} = \frac{c}{k} \qquad \frac{b}{j} = \frac{d}{k}$$

(of course, $j = a + b$ AND $k = c + d$)

Example 2

Find the value of x.

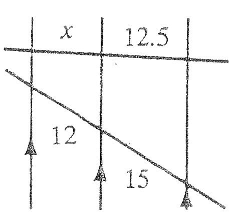


Solution

$$\frac{4}{16} = \frac{x}{25 - x}$$
$$\frac{1}{4} = \frac{x}{25 - x}$$
$$25 - x = 4x$$
$$25 = 5x$$
$$x = 5$$

Find the value of x. Try on your own or with a partner

7.



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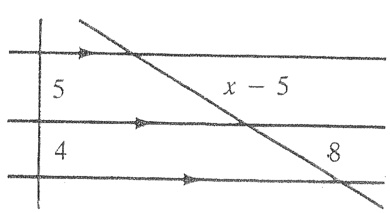

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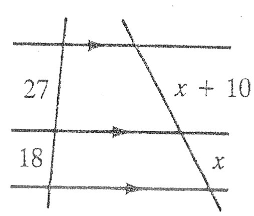
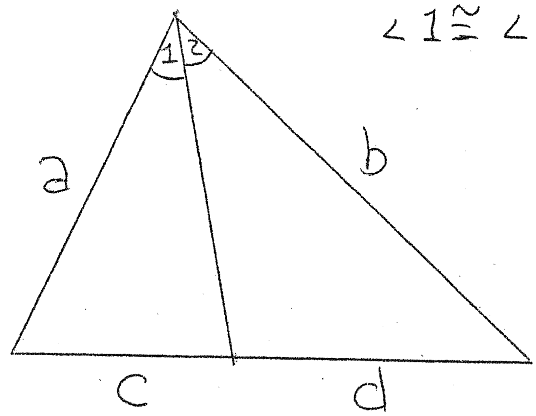


Triangle Angle-Bisector Theorem **If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.**

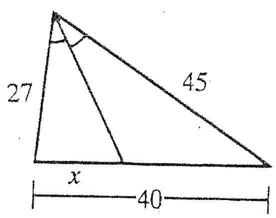


$\angle 1 \cong \angle 2$

$\frac{a}{b} = \frac{c}{d}$ OR $\frac{a}{c} = \frac{b}{d}$

Example 3

Find the value of x.

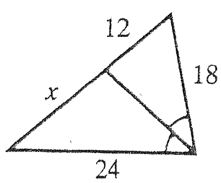


Solution

$$\frac{x}{40 - x} = \frac{27}{45}$$
$$\frac{x}{40 - x} = \frac{3}{5}$$
$$5x = 120 - 3x$$
$$8x = 120$$
$$x = 15$$

Find the value of x. Try these on your own or with a partner

10.



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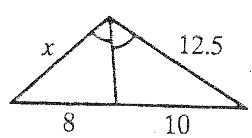

12.